

ALM - Interest Rate Risk Management

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Course Program

- Basic interest rate theory
- Interest rate risk management
- Stochastic term structure models
- Risk measurement
- Reinsurance and insurance-linked securities
- Mean-variance analysis for ALM

Contents of the chapter

- Matching (cash flow matching).
- Immunisation (duration/convexity matching).

Asset-Liability Matching Assumptions

- Assume that the insurance company has an expected **liability cash flow** of $\{L(t) : t > 0\}$ and
- invested in assets which provide a **future cash flow** of $\{A(t) : t > 0\}$.
- they valued both cash flows in its balance sheet by discounting using the **zero-coupon yield curve** $\{y(t) : t > 0\}$:

$$PV_L = \int_0^{\infty} e^{-y(t)t} dL(t); \quad PV_A = \int_0^{\infty} e^{-y(t)t} dA(t)$$

Asset-Liability Matching 1

- Let $PV = PV_A - PV_L$ be the surplus or net asset value.
- The only way to protect the surplus against all changes in the yield curve is by matching both cash flows.
- Assume we have n bonds and that $\mathbf{C}^{n \times n}$ is the payoff matrix of n bonds.
- In theory, a matching portfolio of bonds can be found by solving the system of equations

$$\begin{aligned} (w_1, \dots, w_n) \cdot \mathbf{C} &\stackrel{!}{=} (L_1, \dots, L_n) \\ \Rightarrow (w_1, \dots, w_n) &= (L_1, \dots, L_n) \cdot \mathbf{C}^{-1} \end{aligned}$$

Asset-Liability Matching 2

- The market value of this matching portfolio is then

$$(w_1, \dots, w_n) \cdot \mathbf{B} \stackrel{!}{=} (L_1, \dots, L_n) \cdot \mathbf{C}^{-1} \mathbf{B},$$

where $\mathbf{B}$ is the vector of present values for each bond.

- This is also known as the discounted value of the liabilities.

Example 1

- In the beginning of script S5, we present an example where:
- we consider a liability cash flow of 1M per year during 12 years;
- we find a portfolio of 12 bonds (the same as in our [market assumptions of script S3](#)) that matches it.

Practical problems

- Not enough bonds available for a long-tailed liability cashflow.
- Insufficient market liquidity at some maturities.
- The matching portfolio may have some $w_i < 0$ (inadmissible).
- Investment manager may consider matching portfolio sub-optimal.
- Liabilities are random and change all the time (need for rebalancing).

Cash flow immunization

- Proposed by Frank Redington in 1952.
- Given that full asset-liability matching is not practical, immunisation attempts to give approximately the same interest rate sensitivity to the assets and liabilities.
- Let $PV = PV_A - PV_L$ be the surplus or net asset value to be protected from changes in the yield curve.
- The dollar duration and dollar convexity of the surplus are:

$$DD = PV_A \cdot D_A - PV_L \cdot D_L$$

$$DC = PV_A \cdot C_A - PV_L \cdot C_L$$

Immunization - first order matching 1

- A parallel shift of $\Delta\bar{y}$ in the yield curve will change the net asset value by approximately

$$\Delta PV \approx -DD \cdot \Delta\bar{y} = (-PV_A \cdot D_A + PV_L \cdot D_L) \Delta\bar{y}.$$

- Here we use the **first term** of the **Taylor expansion**.
- To protect the value of its surplus against a **shift** in the yield curve, we could select its assets in such a way that

$$DD_A = PV_A \cdot D_A = PV_L \cdot D_L = DD_L.$$

- This is known as **immunization** or **duration matching**.

Immunization - first order matching 2

- Summing up, **immunization** means that the dollar duration of assets equals the dollar duration of liabilities.
- If

$$PV_A = PV_L$$

then we must make sure that the durations are also equal,

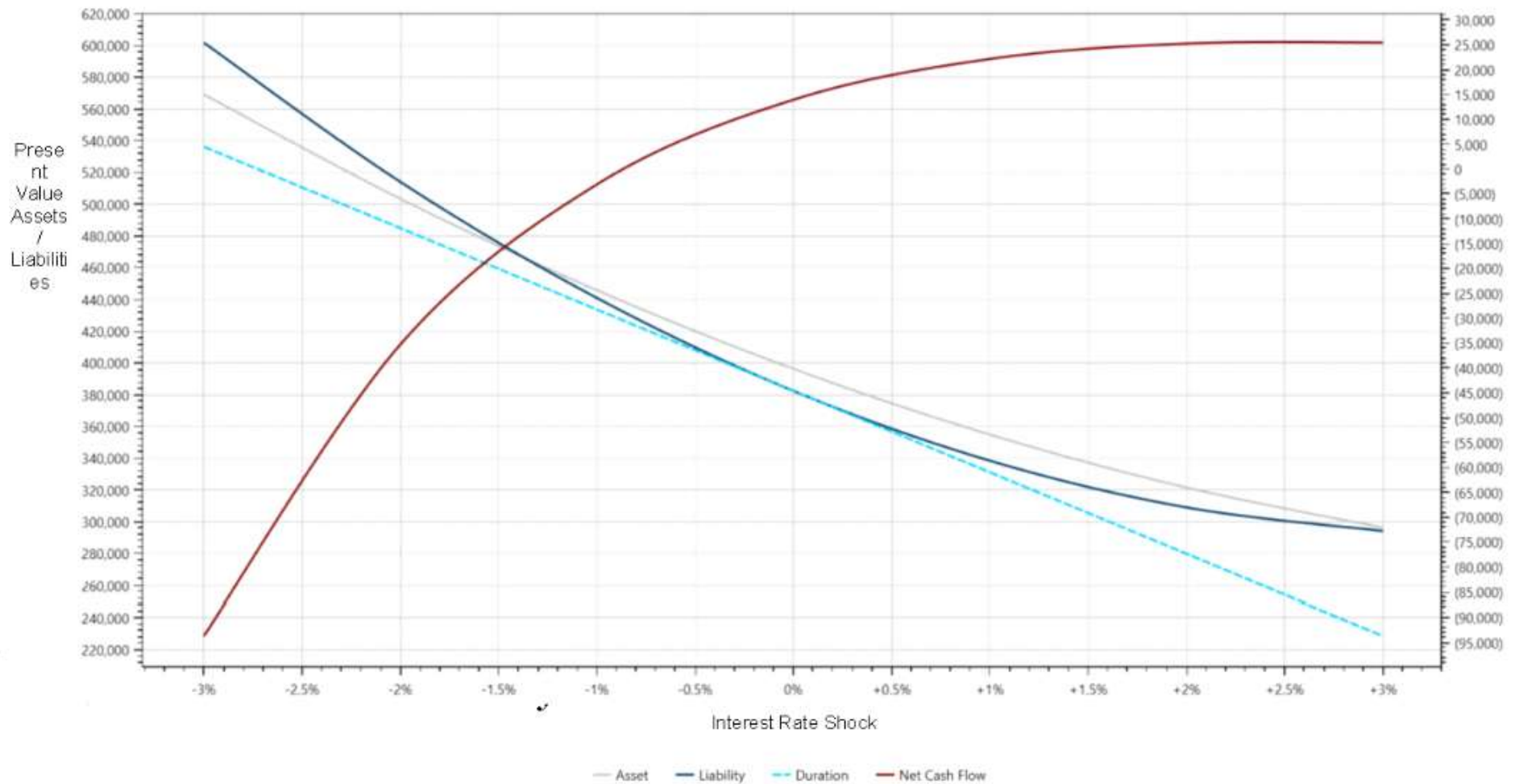
$$D_A = D_L,$$

in order to achieve first order immunization.

Immunization - second order matching

- Assets with **higher convexity** experience **smaller** price declines when yields **rise** and **larger** price gains when yields **fall**.
- Convexity measures the **sensitivity of the duration**.

Convexity Exposure



Some Linear Algebra

- Any liability cash flow can be **duration**-immunised with **two** asset cash-flows by solving the set of linear equations that equate
 1. the dollar present value
 2. the dollar duration
- Any liability cash flow can be **duration and convexity** immunised with **three** asset cash flows, by solving the set of linear equations that equate
 1. the dollar present value
 2. the dollar duration **and** the dollar convexity.

Example 2

- In the remainder of script 5, we present an example where:
- we consider the same liability cash flow of 1M per year during 15 years;
- we find a portfolio of 3 given bonds that achieves immunization.
- we check its resilience against perturbations in the yield curve (homework).

Alternatives to immunization

- Alternatives between the extremes of complete matching and total independence, of asset and liability values:
 - Immunising with “maturity buckets” of bonds gives a **less spiky** asset cash flow.
 - Immunising separate **maturity sections** of the liability cash flow gives better protection.
 - Not immunising but **limiting** the **dollar duration** of the surplus: $PV_A D_A - PV_L D_L \leq \epsilon \cdot PV_A$
 - Make sure the assets have a **larger dollar convexity** than the liabilities: $PV_A C_A > PV_L C_L$